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14. Abstract/Notes

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15. Remarks

RELATIVISTIC ELECTROMAGNETIC WAVES

IN AN ELECTRON-ION PLASMA

by

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ABSTRACT

High power laser beams can drive plasma particles to relativistic energies. An accurate description of strong waves requires the inclusion of ion dynamics in the analysis. The equations governing the propagation of relativistic electromagnetic waves in a cold electron-ion plasma can be reduced to two equations expressing conservation of energy-momentum of the system. The two conservation constants are functions of the plasma stream velocity, the wave velocity, the wave amplitude and the electron-ion mass ratio. The dynamic parameter, expressing electron-ion momentum conservation in the laboratory frame, can be regarded as an adjustable quantity, a suitable choice of which will yield self-consistent solutions when other plasma parameters have been specified. Circularly polarized electromagnetic waves and electrostatic plasma waves are used as illustrations.

INTRODUCTION

Recently, there has been considerable interest in the subject of strong electromagnetic waves in plasma because of its applications. to laser-plasma interaction and pulsar electrodynamics[1-3].

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In the presence of an intense laser field, the plasma particles can acquire quiver oscillation energies exceeding particle rest mass energies such that relativistic effects of particle mass variation have to be taken into account[4]. Several studies have demonstrated that relativistic effects are of significant importance in such laser-plasma phenomena as resonant absorption[5], soliton formation[6], self-focusing[7], density profile modification[8] and production of large DC magnetic field[9].

In earlier works on strong traveling waves (see e.g. Ref. 10-12) only the electron motion was considered, whereas the motion of positive ions was ignored. The results obtained thus apply to comparatively small laser intensities for which the ion dynamics is negligible because of its large rest mass. Nevertheless, when a very intense laser field is present, ions can attain considerable quiver velocities. For instance, it has been suggested that the ion dynamics may be important in the later stages of the resonant absorption process[5]; in a recent study of the inverse Faraday effect[13] it was shown that the DC magnetic field induced by a circularly polarized laser can be greatly reduced by the effects of ion motion. Therefore, an accurate description of strong waves requires the inclusion of ion dynamics.

In the theory of relativistic electromagnetic waves in an electron-ion plasma a dynamic parameter, relating electron and ion momenta, necessarily appears. In previous papers (see e.g. Ref. 14), strong wave solutions were obtained by treating the dynamic parameter either as an arbitrary constant or as a plasma drift velocity. This leads to mathematically correct, but physically misleading solutions.

The purpose of this paper is to elucidate the role played by the dynamic parameter in the determination of self-consistent strong wave solutions. It is shown that, the problem can be reduced to two conservation relations for the energy-momentum of the system. In the laboratory frame, the dynamic parameter expresses conservation of electron-ion momenta. In addition, there is an amplitude parameter that expresses conservation of electromagnetic field-particle energy density. These two parameters are functions of the plasma stream velocity, the wave velocity, the wave amplitude and the electron-ion mass ratio. We will conclude that the dynamic parameter must be regarded as an adjustable quantity, equivalent to an eigenvalue of the system, a suitable choice of which will yield self-consistent wave solutions when the value of the amplitude parameter and other plasma parameters have been allocated.

CONSERVATION RELATIONS IN THE LABORATORY FRAME

Consider a cold two-fluid electron-ion plasma. The laboratory frame is chosen to be an arbitrary frame in which a traveling wave propagates with a constant velocity $c\hat{z}/n$ [where c = speed of light, n = index of refraction and $\hat{z} = (o,o,1)$], the plasma moves with a certain stream velocity $\vec{V}_s$ (where $-c < V_s < c$), and the solutions depend on space and time coordinates only through the combination $\theta = t - nz/c$.

The governing equations written in terms of the phase θ are the relativistic equations of motion

$$(1 - nv_{\alpha z}/c) \frac{d}{d\theta} (\gamma_\alpha \vec{v}_\alpha) = \frac{q_\alpha}{m_\alpha} \left(\vec{E} + \frac{\vec{v}_\alpha \times \vec{B}}{c} \right) \quad , \tag{1}$$

the equations of continuity

$$\frac{dN_\alpha}{d\theta} - \frac{n}{c} \frac{d}{d\theta} (N_\alpha v_{\alpha z}) = 0 \quad , \tag{2}$$

and Maxwell's equations

$$- \frac{n}{c} \frac{dE_z}{d\theta} = 4\pi e(N_i - N_e) \quad , \tag{3}$$

$$\frac{dB_z}{d\theta} = 0 \quad , \tag{4}$$

$$n\hat{z} \times \frac{d\vec{E}}{d\theta} = \frac{d\vec{B}}{d\theta} \quad , \tag{5}$$

$$-n\hat{z} \times \frac{d\vec{B}}{d\theta} = \frac{d\vec{E}}{d\theta} + 4\pi e(N_i \vec{v}_i - N_e \vec{v}_e) \quad , \tag{6}$$

where $\gamma_\alpha = (1 - v_\alpha^2/c^2)^{-\frac{1}{2}}$ and $\alpha = (e,i)$.

Conditions describing the average particle number density and flux can be obtained by taking the phase-average of Eqs. (3) and (6), which gives

$$<N_e> = <N_i> \quad , \tag{7}$$

$$<N_e \vec{v}_e> = <N_i \vec{v}_i> \quad , \tag{8}$$

where the angular bracket denotes averaging over one period in θ. In nonlinear theory, it is convenient to define the plasma stream velocity as the ratio of the average particle flux to the average number density[1,15]. It follows from Eqs. (7) and (8) that the electron and ion streaming velocities are equal

$$\vec{V}_s = \frac{<N_e \vec{v}_e>}{<N_e>} = \frac{<N_i \vec{v}_i>}{<N_i>} \quad . \tag{9}$$

A stationary plasma corresponds to the particular case in which $<N_\alpha \vec{v}_\alpha> = 0$.

It is easy to show that $\vec{V}_s$, as defined by Eq. (9), Lorentz-transforms like any velocity. Suppose there are two frames S' and S, where S has a velocity $V\hat{z}$ relative to S'. Then, the averaged particle flux and number densities in two frames are related by

$$<N\vec{v}_\perp> = <N'\vec{v}_\perp'> \quad , \quad <Nv_z> = \Gamma(<N'v_z'> - V<N'>) \; ; \tag{10}$$

$$<N> = \Gamma(<N'> - V<N'v_z'>/c^2) \quad , \tag{11}$$

where $\perp = (x,y)$ and $\Gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$. Note that phase averaging is a Lorentz-invariant operation. Dividing Eq. (10) by Eq. (11) gives

$$\vec{V}_{s\perp} = \frac{\vec{V}_{s\perp}'}{\Gamma(1 - VV_{sz}'/c^2)} \quad , \quad V_{sz} = \frac{V_{sz}' - V}{1 - VV_{sz}'/c^2} \quad , \tag{12}$$

which indeed satisfies the Lorentz transformation for velocities. This suggests that if in S' the plasma has a certain stream velocity $\vec{V}_s'$, then an observer in S that has a velocity V_s' relative to S' will "see" a stationary plasma with $\vec{V}_s = 0$. Hence, the nonlinear dispersion relation for a streaming plasma can be obtained from the nonlinear dispersion relation for a stationary plasma by a Lorentz transformation[12,15,16].

The behavior of electron and ion number densities follows upon integrating Eq. (2), giving

$$N_\alpha = \frac{N^*}{1 - nv_{\alpha z}/c} \quad , \tag{13}$$

where $N^* = <N_\alpha> (1 - nV_{sz}/c)$.

An integration of Eq. (5) gives

$$\vec{B} = n\hat{z} \times \vec{E} + <\vec{B}> \qquad , \qquad (14)$$

where $<\vec{B}>$ denotes the magnetostatic field. In this paper an unmagnetized plasma is considered so $<\vec{B}> = 0$.

A conservation relation for electron and ion momenta can be obtained from the transverse and longitudinal components of Eq. (1), namely[17],

$$\vec{u}_{i\perp} + \mu\vec{u}_{e\perp} = \vec{D}_\perp \qquad , \qquad (15)$$

$$(u_{iz} - n\gamma_i) + \mu(u_{ez} - n\gamma_e) = D_z \qquad , \qquad (16)$$

where $\vec{u} = \gamma\vec{v}/c$, $\mu = m_e/m_i$ and $\vec{D}$ is a constant vector. Eqs. (15) and (16) can be combined into a single equation

$$\vec{u}_i + \mu\vec{u}_e - n(\gamma_i + \mu\gamma_e)\hat{z} = \vec{D} \qquad . \qquad (17)$$

Thus, the dynamic parameter $\vec{D}$ expresses conservation of electron-ion momenta. In the absence of a wave, $\vec{v}_e = \vec{v}_i = \vec{V}_s$, Eq. (17) becomes

$$(1 + \mu)(1 - V_s^2/c^2)^{-\frac{1}{2}}\vec{V}_s - n(1 + \mu)(1 - V_s^2/c^2)^{-\frac{1}{2}}\hat{z} = \vec{D}, \qquad (18)$$

which shows that $\vec{D}$ is a function of $\vec{V}_s$, n and μ. It will be seen later that for large wave amplitudes $\vec{D}$ also depends on the wave intensity. In linear theory, $\vec{v}_\alpha = \vec{V}_s + \vec{v}_{p\alpha}$ (where $\vec{v}_{p\alpha}$ denotes small perturbations), Eq. (17) reduces to

$$\vec{v}_{ip} + \mu\vec{v}_{ep} = 0 \qquad . \qquad (19)$$

Hence, in this limit, $\vec{D}$ is still given by Eq. (18) because the average values of the perturbed particle velocities are zero.

An energy conservation relation can be obtained as follows. A scalar product of Eq. (1) with $\vec{v}_\alpha/c^2$ gives

$$m_e c^2 \frac{d\gamma_e}{d\theta} = -\frac{eN_e}{N^*}\vec{E} \cdot \vec{v}_e \qquad , \qquad (20)$$

$$m_i c^2 \frac{d\gamma_i}{d\theta} = \frac{eN_i}{N^*}\vec{E} \cdot \vec{v}_i \qquad ; \qquad (21)$$

where Eq. (13) has been applied. Adding Eqs. (20) and (21), then making use of Eqs. (5) and (6), it yields

$$m_e c^2 \frac{d\gamma_e}{d\theta} + m_i c^2 \frac{d\gamma_i}{d\theta} = - \frac{1}{8\pi N^*} \frac{d}{d\theta} (E^2 + B^2) \quad . \tag{22}$$

An integration of Eq. (22) then yields a conservation relation for the electromagnetic field-particle energy density

$$\frac{E^2 + B^2}{8\pi} + N^* m_e \gamma_e c^2 + N^* m_i \gamma_i c^2 = N^* m_e c^2 W \quad , \tag{23}$$

where W is a scalar constant. The energy constant W can be considered an amplitude parameter that characterizes the magnitude of wave intensity, as will be shown later. In the absence of a wave, Eq. (23) reduces to

$$W = (1 + 1/\mu) (1 - V_s^2/c^2)^{-\frac{1}{2}} \quad . \tag{24}$$

W must exceed the above value for a wave solution to exist.

CIRCULARLY POLARIZED ELECTROMAGNETIC WAVES

Purely transverse, circularly polarized electromagnetic waves have been studied extensively (see e.g. Ref. 1, 17-19). For the case of an unmagnetized electron-ion plasma streaming in the wave direction, the dispersion relation in the laboratory frame is.

$$n^2 = 1 - \left[\frac{\omega_{pe}^2}{(1 + \nu^2)^{\frac{1}{2}}} + \frac{\omega_{pi}^2}{(1 + \mu^2 \nu^2)^{\frac{1}{2}}} \right] \frac{(1 - V_s^2/c^2)^{\frac{1}{2}}}{\omega^2} \quad , \tag{25}$$

where $\omega_{p\alpha}^2 = 4\pi <N_\alpha> e^2/m_\alpha$ and $\nu = eE_{max}/m_e \omega c$ is an invariant parameter that measures the wave amplitude. Eq. (25) relates the wave velocity, the plasma stream velocity, the electron-ion mass ratio and the wave amplitude. The aim of this paper is to demonstrate that similar dispersion relations for other strong waves can also be obtained through a proper treatment of the conservation parameters $\vec{W}$ and $\vec{D}$.

ELECTROSTATIC PLASMA WAVES

As an illustration, consider the case of electrostatic plasma waves. For longitudinal waves, $\vec{u}_\alpha = (0,0,u_\alpha)$ and the two conservation relations Eqs. (17) and (23) reduce, respectively, to

$$(u_i - n\gamma_i) + \mu(u_e - n\gamma_e) = D \qquad , \qquad (26)$$

$$\frac{1}{2}\left(\frac{du_e}{d\tau}\right)^2 = \frac{W - \gamma_e - \gamma_i/\mu}{(1 - nu_e/\gamma_e)^2} \qquad , \qquad (27)$$

where $\tau^2 = \omega_{pe}^2\theta^2(1 - nV_s/c)$. Evidently, Eq. (27) indicates that an oscillatory solution exists, with a period of oscillation given by

$$P = \frac{\sqrt{2}}{\omega_{pe}} \int_{u_1}^{u_2} \frac{1 - nu_e/\gamma_e}{(W - \gamma_e - \gamma_i/\mu)^{\frac{1}{2}}} du_e \qquad , \qquad (28)$$

where the turning points $u_{1,2}$ are determined by the equation $\gamma_e + \gamma_i/\mu = W$. Note that γ_i is related to u_e and γ_e through Eq. (26). The general dispersion relation (28) recovers the electron plasma result [12] if the ion term γ_i/μ is ignored.

Before discussing the general solutions of Eq. (28), consider first the phenomenon of wavebreaking which may occur for subluminous waves and is relevant to the laser-plasma interaction[5]. A condition for the occurrence of wavebreaking in the laboratory frame can be obtained as follows. First notice from Eq. (13) that, for a stationary plasma, a traveling wave solution exists (i.e. $0 < N_\alpha < \infty$) only if the condition

$$v_\alpha < c/n \qquad\qquad (29)$$

is satisfied. Upon transformation from the stationary plasma case to the streaming plasma case[12] the condition (29) then becomes

$$\begin{cases} v_c < v_\alpha < c/n & \text{if} \quad V_s < c/n \\[2mm] c/n < v_\alpha < v_c & \text{if} \quad V_s > c/n \end{cases} \qquad , \qquad (30)$$

where

$$\frac{v_c}{c} = \frac{1 + V_s^2/c^2 - 2nV_s/c}{2V_s/c - n(1 + V_s^2/c^2)}$$

Hence, for subluminous longitudinal waves, the electron and ion velocities are subject to the conditions (29) and (30), whose violation implies the occurrence of wavebreaking. A graphical display of Eq. (30) in Fig. 1 shows that, for a given wave velocity,
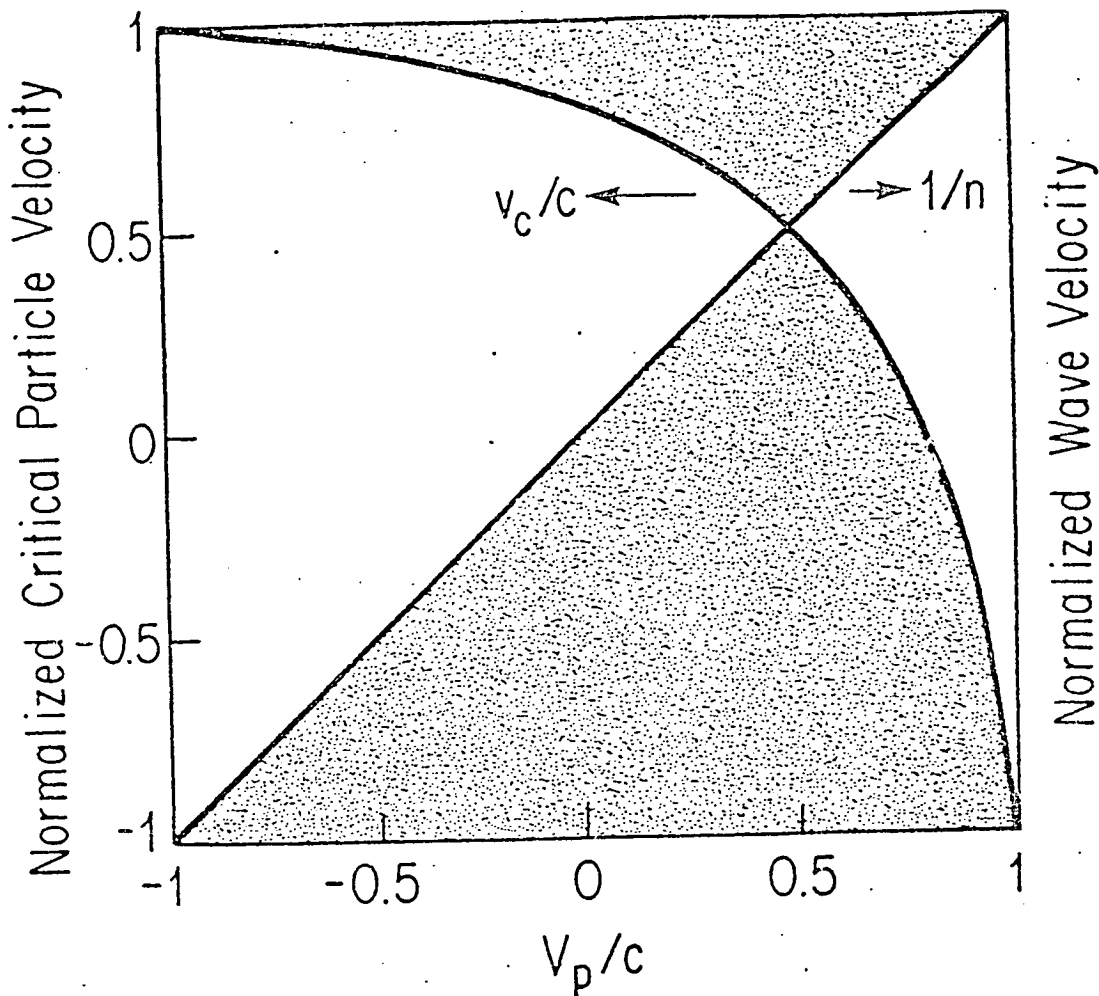
Figure 1. Variation of v_c/c and $1/n$ with normalized wave velocity for $V_s/c = 0.5$. Shaded region indicates wavebreaking.

the range of v_α/c permitted for traveling wave solutions is bounded by the $1/n$ curve and the v_c/c curve. The shaded region represents the domain in which wavebreaking takes place.

The problem of wavebreaking can further be clarified by examining the conservation relations in the wave frame, for which there is no time dependence. In the wave frame, the governing equations become

$$v_\alpha \frac{d}{dz} (\gamma_\alpha v_\alpha) = \frac{q_\alpha}{m_\alpha} E \quad , \tag{31}$$

$$N_e v_e + N_i v_i \equiv N_o V_o = \text{constant} \quad , \tag{32}$$

$$\frac{dE}{dz} = 4\pi e(N_i - N_e) \qquad (33)$$

The above system of equations can be combined to give two conservation relations[20]

$$m_e \gamma_e c^2 + m_i \gamma_i c^2 = m_e c^2 D_1 \qquad (34)$$

$$N_o V_o m_e \gamma_e v_e + N_o V_o m_i \gamma_i v_i - \frac{E^2}{8\pi} = N_o V_o m_e c W_1 \qquad (35)$$

Eq. (34) corresponds to Eq. (26) in the laboratory frame with $D_1 = -(1 - 1/n^2)^{-\frac{1}{2}} D/n$. Therefore, the dynamic parameter expresses
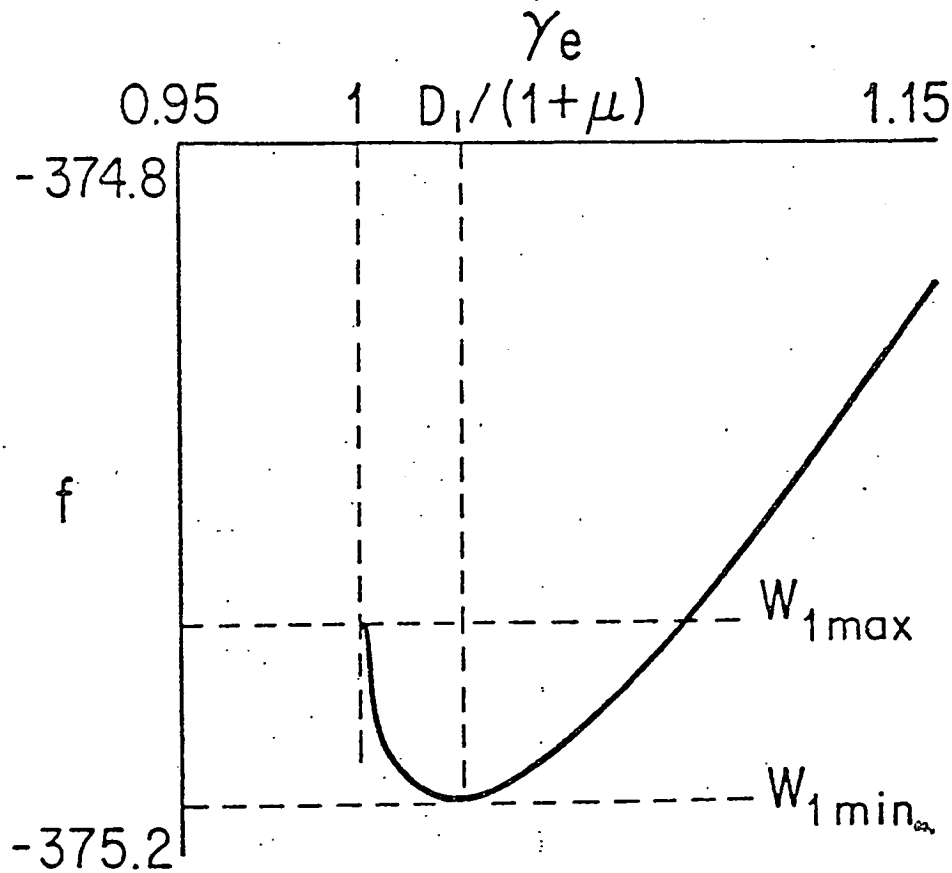

Figure 2. Variation of f with γ_e for $D_1 = 1.021$ (= equilibrium value of D_1, $n = 5$, $\mu = 1/1837$, $V_s = 0$).

conservation of electron-ion energy in the wave frame. Eq. (35) corresponds to Eq. (27) in the laboratory frame and can be rewritten as

$$\frac{1}{2} \left(\frac{d\gamma_e}{d\tau} \right)^2 = W_1 - f(\gamma_e) \qquad , \qquad (36)$$

with

$$f(\gamma_e) = -(\gamma_e^2 - 1)^{\frac{1}{2}} - \frac{1}{\mu} \left[(\mu\gamma_e - D_1)^2 - 1 \right]^{\frac{1}{2}} \quad , \qquad (37)$$

where $\tau_1 = z\omega_{pel}(|V_o|/c^3)^{\frac{1}{2}}$, $\omega_{pel} = 4\pi N_o e^2/m_e$ and $W_1 = -(1 - 1/n^2)^{\frac{1}{2}}W$. Hence, the amplitude parameter expresses conservation of electro-magnetic field-particle momentum density in the wave frame. A typical plot of f as a function of γ_e is displayed in Fig. 2,
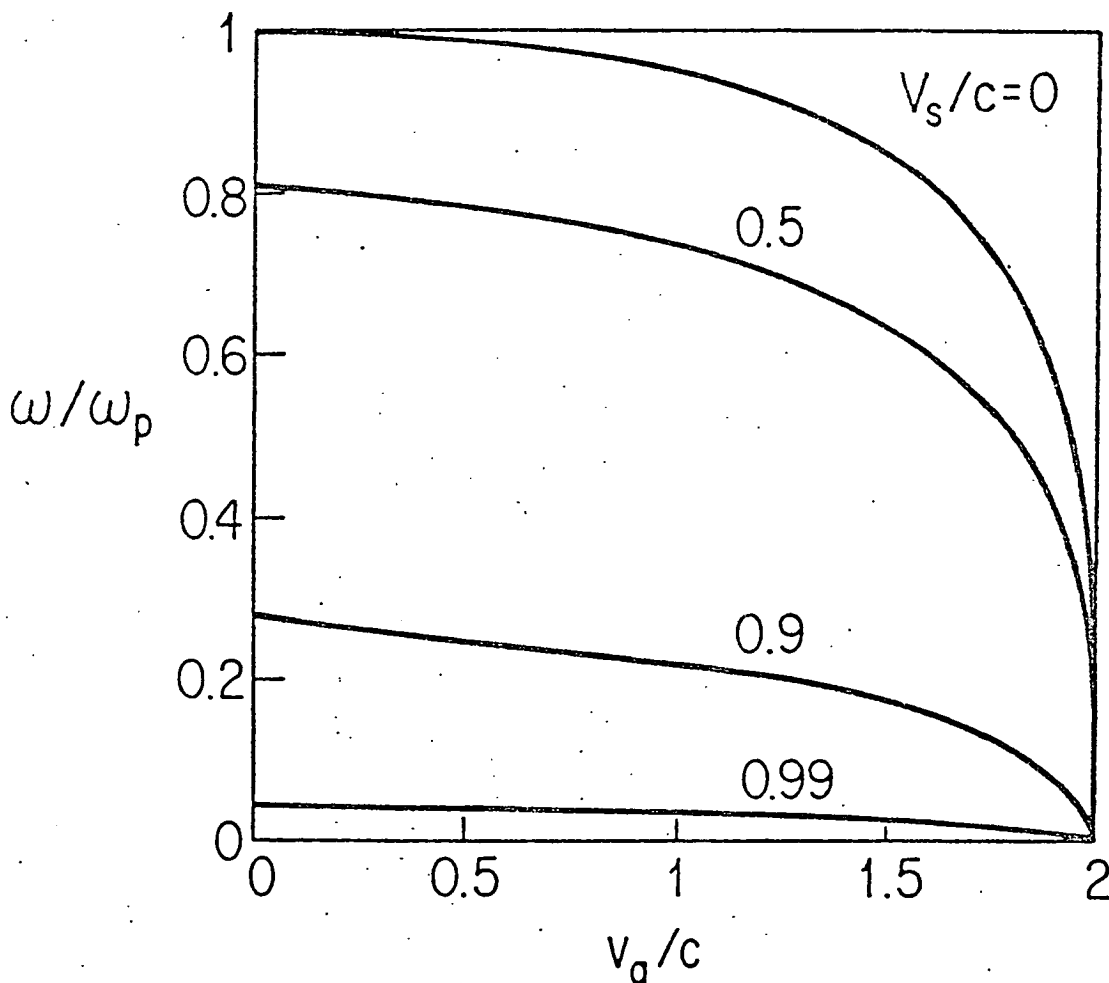

Figure 3. Variation of ω/ω_p with v_a/c for indicated values of V_S/c; $n = 0$.

which shows that oscillatory solutions exist only if W_1 lies in the interval

$$W_{min} < W < W_{max} \tag{38}$$

where $W_{min} = -[D_1^2 - (1 + \mu)^2]^{\frac{1}{2}}/\mu$ and $W_{max} = -[(\mu - D_1)^2 - 1]^{\frac{1}{2}}/\mu$. It is easy to see from Fig. 2 that wavebreaking occurs if W_1 exceeds W_{max}. Furthermore, it shows that W_1 determines the amplitude of the wave oscillations.

The general properties of electrostatic plasma waves in the laboratory frame can be computed from Eq. (28). When evaluating a particular solution it is necessary to decide what values should be assigned to the two conservation parameters W and D. Our wave frame analysis indicates that W can be considered as an amplitude parameter. To obtain a specific solution with given plasma stream velocity, wave velocity, the electron-ion mass ratio and the wave amplitude, the dynamic parameter D must be adjusted accordingly so as to render self-consistency to the solution. Fig. 3 shows the variation of the cut-off frequency ω/ω_p (where $\omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2$ and $n = 0$) with v_α/c (where $v_\alpha = |v_1 - v_2|$, $v_{1,2}$ being the turning points of v_e oscillations) for different values of plasma stream velocity. The computations are carried out for a hydrogen plasma ($\mu = 1/1837$). In Fig. 4, the variation of the self-consistent dynamic parameter D with u_α (where $u_\alpha = |u_1 - u_2|$) for a stationary plasma is displayed for different values of n. It confirms that the dynamic parameter is a function of wave amplitude. Note that for small wave amplitudes, D stays very close to the value given by Eq. (18).

CONSERVATION RELATIONS IN THE SPACE-INDEPENDENT FRAME

For superluminous electromagnetic waves the analysis can be simplified by referring to the space-independent frame which has a velocity $nc\hat{z}$ with respect to the laboratory frame. The basic equations in this frame are

$$\frac{d(\gamma_\alpha \vec{v}_\alpha)}{d\tau} = \frac{q_\alpha}{m_\alpha} \vec{E} \tag{39}$$

$$N_e = N_i \equiv N = \text{constant} \tag{40}$$

$$\vec{B} = \text{constant} \equiv 0 \tag{41}$$

$$\frac{d\vec{E}}{d\tau} + 4\pi Ne(\vec{v}_i - \vec{v}_e) = 0 \tag{42}$$

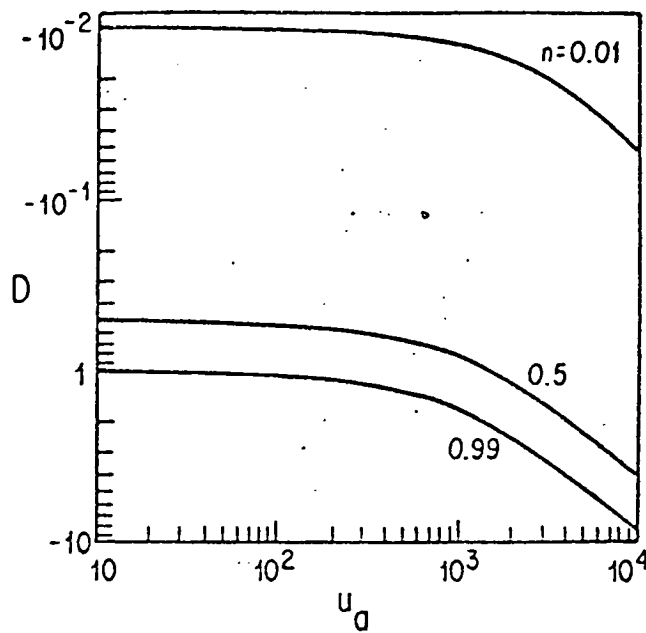The above system of equations can be combined to yield two conser-

Figure 4. Variation of D with u_α for indicated values of n;
$V_s = 0$.

vation relations[18]

$$\begin{cases} m_e \gamma_e \vec{v}_e + m_i \gamma_i \vec{v}_i = m_e c\, \vec{D}_2 & , \qquad (43) \\[2em] \dfrac{E^2}{8\pi} + N m_e \gamma_e c^2 + N m_i \gamma_i c^2 = N m_e c^2 W_2 & . \qquad (44) \end{cases}$$

Eq. (43) corresponds to Eq. (17) in the laboratory frame. Upon transforming to the laboratory frame, the transverse components of Eq. (43) gives Eq. (15) with $\vec{D}_{2\perp} = \vec{D}_\perp$, while the longitudinal component of Eq. (43) gives Eq. (16) with $D_{2z} = (1 - n^2)^{-\frac{1}{2}} D_z$. Hence the dynamic parameter expresses conservation of electron-ion momentum in the space-independent frame. Eq. (44) corresponds to Eq. (25) in the laboratory frame and can be rewritten as

$$\frac{1}{2}\left(\frac{d\vec{u}_e}{d\tau_2}\right)^2 + \gamma_e + \gamma_i/\mu = W_2 \qquad , \qquad (45)$$

where $\tau_2 = \omega_{pe2}t$, $\omega_{pe2}^2 = 4\pi Ne^2/m_e$ and $W_2 = (1 - n^2)^{\frac{1}{2}}W$. Therefore, the amplitude parameter expresses conservation of electromagnetic field-particle energy density in the space-independent frame. For the special case of $\vec{D}_2 = D_2\hat{z}$, W_2 must exceed $[D_2^2 + (1 + \mu)^2]^{\frac{1}{2}}/\mu$ for a solution to exist[18]. An elegant formulation of the energy-momentum conservation relations in terms of the Maxwell's stress tensor in the space-independent frame can be found in Ref. 21.

The two energy-momentum relations (43) and (45) can be used to study all possible wave solutions. As in the case of electrostatic plasma oscillations, a proper handling of the conservation constants, $\vec{D}_2$ and W_2, is essential for obtaining self-consistent solutions.

The case of circularly polarized electromagnetic waves serves to demonstrate the correct behavior of the two conservation constants. For simplicity, consider the plasma to be stationary in the laboratory frame (i.e., $\vec{V}_s = 0$). For circularly polarized waves, γ_e and γ_i are constants given, in the space-independent frame, by[18]

$$\gamma_e = \left(\frac{1 + \nu^2}{1 - n^2}\right)^{\frac{1}{2}} \quad , \quad \gamma_i = \left(\frac{1 + \mu^2\nu^2}{1 - n^2}\right)^{\frac{1}{2}} \quad (46)$$

where $\nu = eE_{max}/m_e\omega c$ as defined in Eq. (25). Substitution of Eq. (46) into Eqs. (43) and (45), respectively, yields

$$\vec{D}_2 = -n(1 - n^2)^{-\frac{1}{2}}[(1 + \mu^2\nu^2)^{\frac{1}{2}} + \mu(1 + \nu^2)^{\frac{1}{2}}]\hat{z} \quad , \quad (47)$$

$$W_2 = \nu^2/2 + (1 - n^2)^{-\frac{1}{2}}[(1 + \nu^2)^{\frac{1}{2}} + (1 + \mu^2\nu^2)^{\frac{1}{2}}/\mu] \quad . \quad (48)$$

Note that $\vec{D}_{2\perp} = 0$ and $(d\vec{u}_e/d\tau_2)^2 = \nu^2$ for circularly polarized waves. Eqs. (47) and (48) show clearly that the energy-momentum conservation parameters are functions of the wave velocity, the wave amplitude and the electron-ion mass ratio. In addition, from Eqs. (47) and (48) one obtains the following relation

$$\vec{D}_2 = \mu n(\nu^2/2 - W_2)\hat{z} \quad , \quad (49)$$

which shows explicitly that for a given amplitude parameter W_2 there is only one value of the dynamic parameter $\vec{D}_2$ that satisfies self-consistent solutions.

CONCLUSION

It has been shown that the equations governing the propagation of relativistic electromagnetic waves in an electron-ion plasma can

be reduced to two energy-momentum conservation relations. The two
conservation constants, the amplitude parameter and the dynamic
parameter, respectively, are functions of the plasma stream velocity,
the wave velocity, the electron-ion mass ratio and the wave in-
tensity. In order to obtain self-consistent solutions the dynamic
parameter must be chosen appropriately when other parameters have
been specified.

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